Verifying Probabilistic Correctness in Isabelle with pGCL

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NICTA

Australian Government
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Outline

- Stochastic Behaviour in Systems
- Functional vs. Probabilistic Verification
- pGCL in Isabelle/HOL
- Example: Lattice-Lottery Scheduler

Sources of Uncertainty

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- A modern processor.

Probability Refines Nondeterminism

Classical nondeterminism is the ultimate in pessimism: Anything that *can* happen *will* happen.

If we know how events are distributed, we can do better.

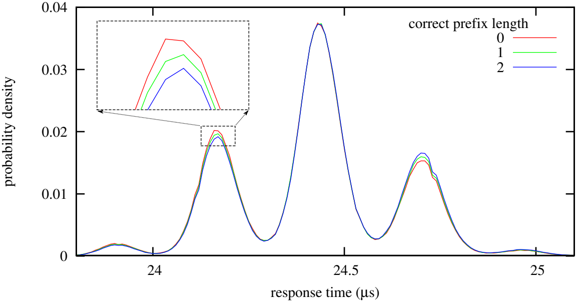
Probabilistic models are a halfway-house between full nondeterminism and full predictability.

Probabilistic guarantees are relevant both for security, and for reliability.

Our current work is on probabilistic security guarantees.

Why is this relevant in systems?

Feed a secret string and a guess to strcmp:



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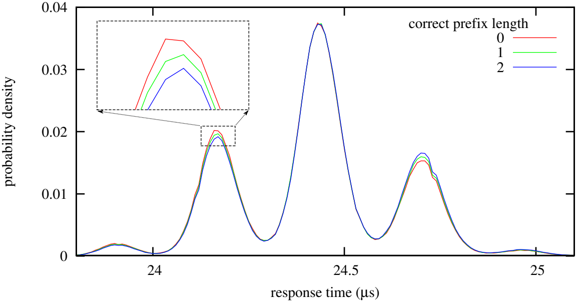

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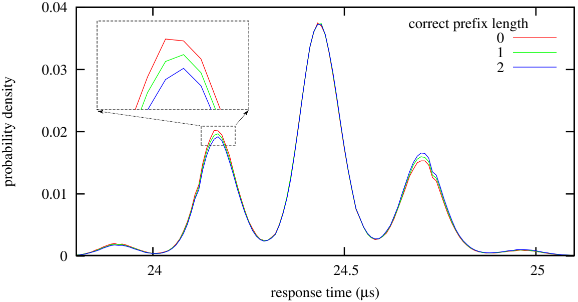

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How bad is it? How can we mitigate it?
How will it behave in a larger system?

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We want to show something like:

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Formulating this rigorously is the subject of our existing work.

Mechanising this work in Isabelle/HOL ensures our reasoning is sound, and scalable to large problems.

We use pGCL, an extension of Dijkstra's GCL with probability.

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$$\wp\ a\ Q \equiv \sup\,\{P | P\ a\ Q\}$$

$$\{R\} \leq \{S\} \equiv R \vdash S \equiv \forall s.\ R\ s \rightarrow S\ s$$

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Thus $P = \{x = 0 \vee x = 1\} \cap \{x = 0\} = \{x = 0\}$.

We are treating annotations as sets.

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These quantitative predicates are called *expectations*.

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$\wp\ (a\ {}_{p}{\oplus}\ b)\ (y = x)\ 0 = 1$ and $\wp\ (a\ {}_{p}{\oplus}\ b)\ (y = x)\ 1 = 1/2$.

All other values are zero.

Combining Probability and Nondeterminism

How about this?

$$E = \wp\left((y := x^2 \ {}_{1/2}\!\oplus\ y := 2x) \sqcap \\ (y := x^2 \ {}_{1/3}\!\oplus\ y := 2x)\right)(y = x)$$

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Simply apply both rules:

$$E\ x = \min\ (1/2 \times «x = 0 \lor x = 1» + 1/2 \times «x = 0») \\ (1/3 \times «x = 0 \lor x = 1» + 2/3 \times «x = 0»)$$

This time, $E\ 0 = 1$ and $E\ 1 = 1/3$.

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$E\ x$ is the *minimum* probability that $y = x$ will hold.

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pGCL

These are basics of pGCL (Morgan & McIver, 2004).

It's a formal model of computation incorporating probability and nondeterminism.

In the remainder of the talk I will introduce our mechanisation in Isabelle/HOL, and our work on the probabilistic verification of systems software.

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The state space need not, in general, be finite.

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$$\forall P\, c\, s.\ (\text{sound}\ P \wedge 0 < c) \longrightarrow c \times t\ P\ s = t\ (\lambda s.\ c \times P\ s)\ s$$

A few primitives

$$\text{Abort} \equiv \lambda ab\, P.\ \text{if } ab \text{ then } \lambda s.\ 0 \text{ else } \lambda s.\ \text{bound\_of } P$$

We model both strict (WP) and liberal (WLP) semantics.

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Embedding a Monad

The shallow embedding makes it easy to embed the L4.verified nondeterministic monad:

$$\text{Exec} :: (\sigma \Rightarrow (\alpha \times \sigma)\ \text{set}) \Rightarrow \text{bool} \Rightarrow (\sigma \Rightarrow \mathbb{R}) \Rightarrow \sigma \Rightarrow \mathbb{R}$$

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We lift Hoare triples to probabilistic entailments:

$$\frac{\{P\}\ prog\ \{\lambda r\ s.\ Q\ s\} \quad \forall s.\ prog\ s \neq \{\} \quad \exists s.\ P\ s}{«P» \vdash \wp\ prog\ «Q»}\ \textsc{wp\_Exec}$$

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In pGCL, an implementation establishes any property with at least as great a probability as its specification.

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The security property:

$$\forall c, n.\ (c, n) \in S \rightarrow \text{sec\_class}.c \leq \text{sec\_class}.n \vee \\ n = \text{downgrader}$$

Unfairness

$$\begin{tikzcd}
 & H & \\
L_a \arrow[ur] & & L_b \arrow[ul] \\
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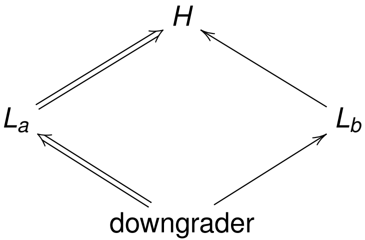

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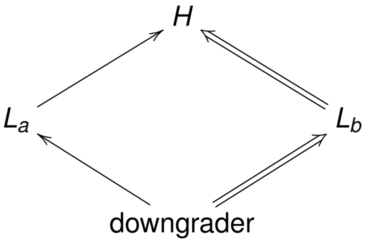

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A nondeterministic scheduler might simply always pick L_b.

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This scheduler is a Markov process, and if T is irreducible and positive recurrent, there exists a stationary distribution.

Lottery Scheduling

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An efficient implementation might use a lottery:

$$
\begin{aligned}
&\text{scheduleM } t \equiv \text{do} \\
&\quad c \leftarrow \text{gets cd};\ \ l \leftarrow \text{gets lottery}; \\
&\quad \text{let } n = l\ c\ t \text{ in modify}(\lambda s.\ s(\text{cd} := n)) \\
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We chain in probability from above:

$$\text{scheduleC} \equiv t \text{ from UNIV at } 2^{-32} \text{ in Exec (scheduleM } t)$$

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Data Refinement

We cannot show that scheduleR $\sqsubseteq$ scheduleC, as they operate on different state spaces:

$$\textbf{record}\ \text{stateA} = \mathit{cd} :: \text{domain}$$

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Take the natural projection: $\phi :: \text{stateC} \Rightarrow \text{stateA}$.

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The kernel may modify the lottery!

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The kernel implements a fair, secure scheduler.

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Questions?